INDIAN STATISTICAL INSTITUTE, BANGALORE CENTRE
B.MATH - First Year, First Semester, 2010-11
Probability Theory-I, Midterm Examination

1. A balanced die is rolled n times independently where $n \geq 2$. Let X denote the number of times 6 dots show up and Y denote the number of times 5 dots show up in these n rolls.
(a) What is the joint probability distribution of (X, Y)?
(b) Find the probability distribution of $Z = X + Y$.
(c) Find $E(Z)$, $Var(Z)$ and $Cov(X, Z)$. [9]

2. Consider a sequence of independent Bernoulli trials with probability of success p, $0 < p < 1$. Let Y_1 be the number of successes in the first n_1 trials, and Y_2 be the number of successes in the next n_2 trials.
(a) What is the joint probability distribution of (Y_1, Y_2)?
(b) What is the probability distribution of $T = Y_1 + Y_2$?
(c) What is the conditional distribution of Y_1 given $T = t$?
(d) Find $E(Y_1|T = t)$ and $Var(Y_1|T = t)$. [12]

3. Let X be a non-negative integer valued random variable with $p_n = P(X = n)$, $n = 0, 1, 2, \ldots$. Suppose the sequence $\{p_n\}$ satisfies $p_n = \frac{1}{3}p_{n-1}$, $n = 1, 2, \ldots$.
(a) Derive the probability mass (density) function of X.
(b) Find $E\{(X - 2)^2\}$. [10]

4. Assume that there are equal number of males and females in a particular population. Suppose that 5% of men and 1% of women are colour-blind. A colour-blind person is chosen at random. What is the probability of this person being male? [8]

5. Assume that the number of eggs laid by an insect follows a Poisson distribution with parameter λ. Once laid, each egg has probability p of hatching, and the hatching of one egg is independent of the hatching of the others. An entomologist studies a set of n such insects, observing both the number of eggs laid and the number of eggs hatching. Assume that the different insects and their descendants behave independently. Let S denote the total number of eggs hatching in this experiment.
(a) What is the probability distribution of S?
(b) Find $E(S)$ and $Var(S)$.
(c) Consider one of the insects mentioned above. Suppose you are told that the number of eggs hatching (of this insect) is y. Find the probability distribution of the number of eggs laid by this insect. [11]